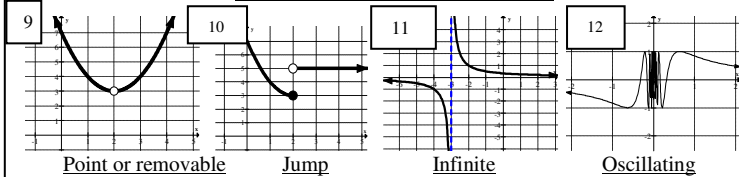


LIMIT LAWS

- $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
- $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$
- $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x} = 0$
- $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$
- $\lim_{x \rightarrow 0} \frac{\sin(ax)}{(bx)} = \lim_{x \rightarrow 0} \frac{\sin(ax)}{\sin(ax)} = \lim_{x \rightarrow 0} \frac{(bx)}{\sin(ax)} = \frac{a}{b}$
- $\lim_{x \rightarrow a} f(x) = L$ (exists) If and only if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = L$
- $f(x)$ is cont at a if $\lim_{x \rightarrow a} f(x) = f(a)$

8. **Continuity at a** if $\lim_{x \rightarrow a^-} f(x) = f(a) = \lim_{x \rightarrow a^+} f(x)$

TYPES OF DISCONTINUITY



ARITHMETIC OF INFINITY

$1. \infty + \infty = \infty$ $2. n + \infty = \infty$ $3. \infty + 0 = \infty$ $4. 0 + \infty = \infty$	(+)	$1. \infty \cdot \infty = \infty$ $2. n \cdot \infty = \infty$ $3. 0 \cdot \infty = 0$	(×)
$1. \infty - \infty = \text{und}$ $2. n - \infty = -\infty$ $3. \infty - n = \infty$ $4. n - n^- = 0^- = -\frac{1}{\infty}$ $5. n^+ - n = 0^+ = \frac{1}{\infty}$	(-)	$1. \infty / \infty = \text{und}$ $2. n / \pm \infty = 0$ $3. \infty / n = \infty$ $4. n / 0 = \pm \infty$ $5. n / 0^+ = \infty$ $6. n / 0^- = -\infty$	(÷)
$1. \infty^\infty = \infty$ $3. \infty^0 = 1$ $4. n^\infty = \infty$	power	$2. \infty^n = \infty$ $5. 0^\infty = 0$	Trig
		$1. -1 \leq \sin(\pm \infty) \leq 1$ $2. -1 \leq \cos(\pm \infty) \leq 1$	

LIMITS(1)

Basic Rules

- $\frac{d}{dx} c = 0$ (Constant)
- $\frac{d}{dx} c[f(x)] = c \frac{d}{dx} f(x)$ (constant multiple)
- $\frac{d}{dx} x^n = nx^{n-1}$ (power)
- $\frac{d}{dx} (u \pm v) = \frac{d}{dx} u \pm \frac{d}{dx} v$ (Sum & Difference)
- $\frac{d}{dx} (uv) = v \frac{d}{dx} u + u \frac{d}{dx} v$ (Product)
- $\frac{d}{dx} \left(\frac{u}{v}\right) = \frac{v \frac{d}{dx} u - u \frac{d}{dx} v}{v^2}$ (Quotient)
- $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$ (Inverse)
- $\frac{d}{dx} f(g(x)) = \frac{d}{dx} f(u) \cdot \frac{d}{dx} g(x)$ where $u = g(x)$ (Chain Rule)

Trigonometric Functions

- $\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$
- $\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$
- $\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$
- $\frac{d}{dx} \cot u = -\csc^2 u \frac{du}{dx}$
- $\frac{d}{dx} \sec u = \sec u \tan u \frac{du}{dx}$
- $\frac{d}{dx} \csc u = -\csc u \cot u \frac{du}{dx}$

Inverse Trigonometric Functions

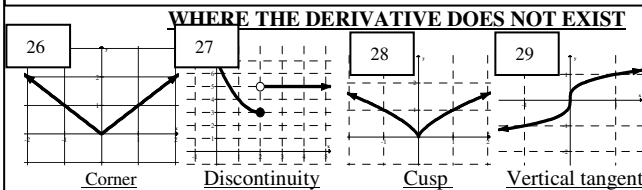
- $\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$
- $\frac{d}{dx} \cos^{-1} u = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}$
- $\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}$
- $\frac{d}{dx} \cot^{-1} u = \frac{-1}{1+u^2} \frac{du}{dx}$
- $\frac{d}{dx} \sec^{-1} u = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$
- $\frac{d}{dx} \csc^{-1} u = \frac{-1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$

DERIVATIVES(2)

Exponential and Logarithmic Functions

- $\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$
- $\frac{d}{dx} e^u = e^u \frac{du}{dx}$
- $\frac{d}{dx} \sqrt{u} = \frac{1}{2\sqrt{u}} \frac{du}{dx}$
- $\frac{d}{dx} \log_a u = \frac{1}{u \ln a} \frac{du}{dx}$
- $\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$
- $\frac{d}{dx} |u| = \frac{u}{|u|} \frac{du}{dx}$

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Linearization: $L(x) = f(a) + f'(a)(x-a)$

30. **Mean Value Theorem:** If f is cont on $[a,b]$ on and diff on (a,b)

$$\Rightarrow \text{exist a } c \in (a,b) \text{ s.t. } f'(c) = \frac{f(b) - f(a)}{b-a}$$

31. **Rolle's Theorem:** MVT where $f'(c) = \frac{f(b) - f(a)}{b-a} = 0$

32. **Intermediate Value Theorem:** If f is cont on $[a,b]$ and $d \in [f(a), f(b)]$ then there is a $c \in [a,b]$ st $f(c) = d$

Definition of Derivative

$$33. f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$34. f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x-a}$$

35. $f'(x) \approx$ Average rate of change
 = Slope of Secant line
 $= \frac{f(b) - f(a)}{b-a}$

APPROXIMATING AREA

- LRAM** $_n = w(f(x_1) + f(x_2) + \dots + f(x_{n-1}))$ or $w_1 f(x_1) + w_2 f(x_2) + \dots + w_{n-1} f(x_{n-1})$
- RRAM** $_n = w(f(x_2) + f(x_3) + \dots + f(x_n))$ or $w_1 f(x_2) + w_2 f(x_3) + \dots + w_{n-1} f(x_n)$
- MRAM** $_n = w \left(f\left(\frac{x_1+x_2}{2}\right) + f\left(\frac{x_2+x_3}{2}\right) + \dots + f\left(\frac{x_{n-1}+x_n}{2}\right) \right)$ or $w_1 f\left(\frac{x_1+x_2}{2}\right) + w_2 f\left(\frac{x_2+x_3}{2}\right) + \dots + w_{n-1} f\left(\frac{x_{n-1}+x_n}{2}\right)$

Note: $w = \frac{b-a}{n}$ and applies only for equal sub intervals

14. $T_n = \frac{w}{2}(y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$ or $\frac{1}{2}(w_1(y_1 + y_2) + w_2(y_2 + y_3) + \dots)$

20. Area = $\int_{x_{left}}^{x_{right}} [f(x)_{top} - f(x)_{down}] dx$ or Area = $\int_{y_{down}}^{y_{top}} [f(y)_{left} - f(y)_{right}] dy$

21. Vol of rev = $\pi \int_{x_{left}}^{x_{right}} \left\{ [f(x)_{top} - a]^2 - [f(x)_{down} - a]^2 \right\} dx$ Vol Cross sect. = $\int_a^b A(x) dx$

ANTIDIFFERENTIATION (INTEGRATION) RULES

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int \frac{1}{ax+b} dx = \frac{\ln|ax+b|}{a} + C$
- $\int e^{kx} dx = \frac{e^{kx}}{k} + C$
- $\int a^x dx = \frac{a^x}{\ln a} + C$
- $\int \sin kx dx = -\frac{\cos kx}{k} + C$
- $\int \cos kx dx = \frac{\sin kx}{k} + C$
- $\int \sec x \tan x dx = \sec x + C$
- $\int \sec^2 x dx = \tan x + C$
- $\int \csc x \cot x dx = -\csc x + C$

15. **Average Value of f** $av(f) = f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$

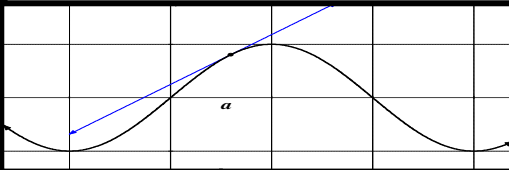
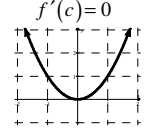
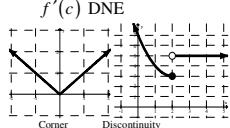
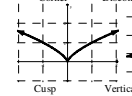

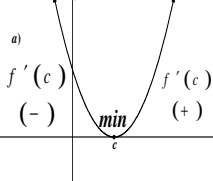
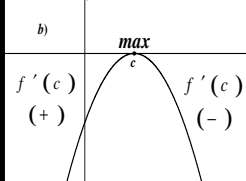
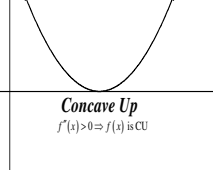
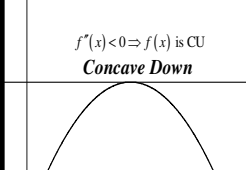
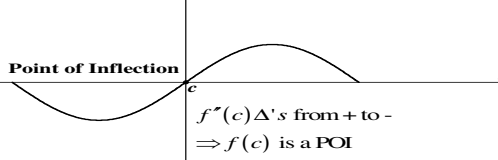
16. FTC I: $\int_a^b f'(x) dx = f(b) - f(a)$ 17. FTC II i) $\int_a^x f(t) dt = F(x)$

ii) $\frac{d}{dx} \int_a^x f(t) dt = f(x)$ iii) $\frac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x)) \cdot g'(x) - h(g(x)) \cdot h'(x)$

18. Integration by parts: $\int v du = uv - \int u dv$ use LIPET to select u

19. Integration by substitution: $\int f(g(x)) g'(x) dx = \int f(u) du$

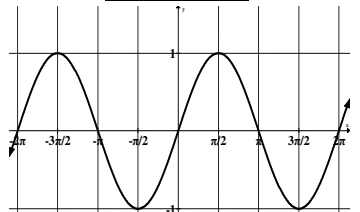
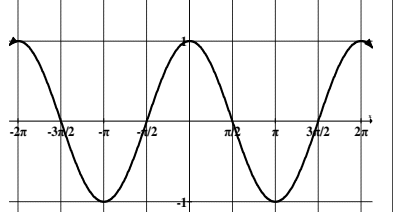
INTEGRALS (3)

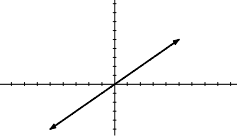
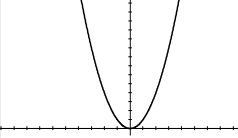
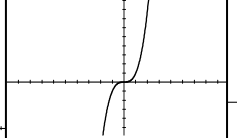
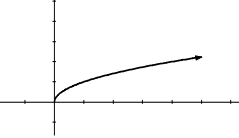
Term	Verbal Description	Symbolic	Graphical
1. Derivative of f at a :	The instantaneous rate of change of the function at a or the slope of the tangent line at a	$f'(a) = \left. \frac{df}{dx} \right _{x=a}$ $= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$	
2. Critical Number c	A number c in an open (a, b) interval where the derivative is zero or does not exist	$c \in (a, b)$ where $f'(c) = 0$ or $f'(c)$ DNE	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $f'(c) = 0$  </div> <div style="text-align: center;"> $f'(c)$ DNE  </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 5px;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div>
3. First Derivative Test	a) If the first derivative changes from negative to positive at c then the function has a relative minimum at c b) If the first derivative changes from positive to negative at c then the function has a relative maximum at c	a) If $f'(c)$ Δ 's from $-$ to $+$ $\Rightarrow f'(c)$ is a min b) If $f'(c)$ Δ 's from $+$ to $-$ $\Rightarrow f'(c)$ is a max	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $f'(c)$ $(-)$  $(+)$ </div> <div style="text-align: center;"> $f'(c)$ $(+)$  $(-)$ </div> </div>
4. Concavity Test	a) If the second derivative is positive on an interval I then the function is Concave Up on I b) If the second derivative is negative on an interval I the function is Concave down on I	a) If $f''(c) > 0$ on I $\Rightarrow f(x)$ is CU on I b) If $f''(c) < 0$ on I $\Rightarrow f(x)$ is CD on I	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p>Concave Up $f''(x) > 0 \Rightarrow f(x)$ is CU</p> </div> <div style="text-align: center;">  <p>$f''(x) < 0 \Rightarrow f(x)$ is CD Concave Down</p> </div> </div>
5. Point of Inflection at c	f : Is a point where the concavity of f changes f' : Is a point where f' changes from increasing to decreasing or decreasing to increasing f'' : Is a point where f'' changes from positive to negative or negative to positive	f Δ 's from CU to CD or CD to CU f' Δ 's from \nearrow to \searrow or \searrow to \nearrow $f''(x)$ Δ 's from $+$ to $-$ or $-$ to $+$	 <p>Point of Inflection $f''(c)$ Δ's from $+$ to $-$ $\Rightarrow f(c)$ is a POI</p>

VOCABULARY (4)

Motion definitions and Equations

6. Displacement: A Vector quantity that represents the net change in position	$s(t) = x(b) - x(a) = \int_a^b v(t)$	7. Distance: A scalar quantity that represents total movement regardless of sign	$d(t) = x(b) - x(a) = \int_a^b v(t) dt$
8. Velocity: A Vector quantity that represents the rate of change of position	$v(t) = s'(t)$	9. Speed: A scalar quantity that represents the rate of covering distance	Speed = $ v(t) $
10. Acceleration: A vector quantity that represents the rate of change of velocity	$a(t) = v'(t) = s''(t)$	11. Given initial position $s(a) = C$ the final position is given by $s(b) = s(a) + \int_a^b s'(t) dt$	

Reciprocal	Quotient	Pythagorean	Sine Curve	Cosine Curve
$\sin x = \frac{1}{\csc x}$ $\csc x = \frac{1}{\sin x}$ $\cos x = \frac{1}{\sec x}$ $\sec x = \frac{1}{\cos x}$ $\tan x = \frac{1}{\cot x}$ $\cot x = \frac{1}{\tan x}$	$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$	$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $\cot^2 x + 1 = \csc^2 x$		

	0	$\pi/6$ (30°)	$\pi/4$ (45°)	$\pi/3$ (60°)	$\pi/2$ (90°)				
sin x	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1	Linear $y = x$	quadratic $y = x^2$	Cubic $y = x^3$	Radical $y = \sqrt{x}$
cos x	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0				
tan x	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Und.				
csc x	Und.	2	$2/\sqrt{2}$	$2/\sqrt{3}$	1				
sec x	1	$2/\sqrt{3}$	$2/\sqrt{2}$	2	Und.				
cot x	Und.	$2/\sqrt{3}$	1	$1/\sqrt{3}$	0	Logarithmic $y = \ln x$	Exponential $y = e^x$	Absolute value $y = x $	circular $y = \sqrt{9 - x^2}$