



**AP<sup>®</sup> Calculus AB**  
**2007 Free-Response Questions**  
**Form B**

**The College Board: Connecting Students to College Success**

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 5,000 schools, colleges, universities, and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT<sup>®</sup>, the PSAT/NMSQT<sup>®</sup>, and the Advanced Placement Program<sup>®</sup> (AP<sup>®</sup>). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

© 2007 The College Board. All rights reserved. College Board, Advanced Placement Program, AP, AP Central, SAT, and the acorn logo are registered trademarks of the College Board. PSAT/NMSQT is a registered trademark of the College Board and National Merit Scholarship Corporation.

Permission to use copyrighted College Board materials may be requested online at:  
[www.collegeboard.com/inquiry/cbpermit.html](http://www.collegeboard.com/inquiry/cbpermit.html).

**Visit the College Board on the Web: [www.collegeboard.com](http://www.collegeboard.com).**

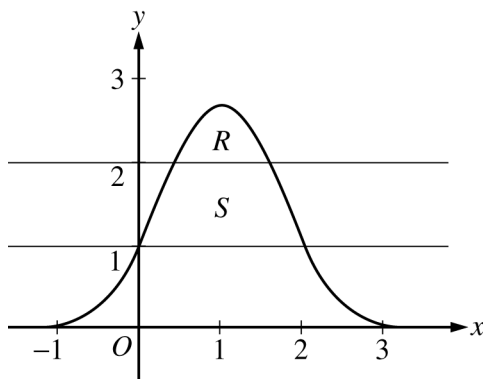
**AP Central is the official online home for the AP Program: [apcentral.collegeboard.com](http://apcentral.collegeboard.com).**

**2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

**CALCULUS AB**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

A graphing calculator is required for some problems or parts of problems.

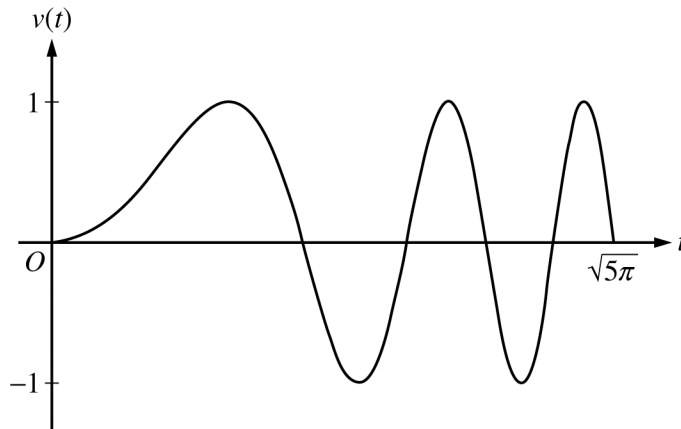
---



1. Let  $R$  be the region bounded by the graph of  $y = e^{2x-x^2}$  and the horizontal line  $y = 2$ , and let  $S$  be the region bounded by the graph of  $y = e^{2x-x^2}$  and the horizontal lines  $y = 1$  and  $y = 2$ , as shown above.
- (a) Find the area of  $R$ .
- (b) Find the area of  $S$ .
- (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 1$ .
- 

**WRITE ALL WORK IN THE EXAM BOOKLET.**

**2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**



2. A particle moves along the  $x$ -axis so that its velocity  $v$  at time  $t \geq 0$  is given by  $v(t) = \sin(t^2)$ . The graph of  $v$  is shown above for  $0 \leq t \leq \sqrt{5\pi}$ . The position of the particle at time  $t$  is  $x(t)$  and its position at time  $t = 0$  is  $x(0) = 5$ .
- (a) Find the acceleration of the particle at time  $t = 3$ .
  - (b) Find the total distance traveled by the particle from time  $t = 0$  to  $t = 3$ .
  - (c) Find the position of the particle at time  $t = 3$ .
  - (d) For  $0 \leq t \leq \sqrt{5\pi}$ , find the time  $t$  at which the particle is farthest to the right. Explain your answer.
- 

**WRITE ALL WORK IN THE EXAM BOOKLET.**

**2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

3. The wind chill is the temperature, in degrees Fahrenheit ( $^{\circ}\text{F}$ ), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity  $v$ , in miles per hour (mph). If the air temperature is  $32^{\circ}\text{F}$ , then the wind chill is given by  $W(v) = 55.6 - 22.1v^{0.16}$  and is valid for  $5 \leq v \leq 60$ .
- (a) Find  $W'(20)$ . Using correct units, explain the meaning of  $W'(20)$  in terms of the wind chill.
- (b) Find the average rate of change of  $W$  over the interval  $5 \leq v \leq 60$ . Find the value of  $v$  at which the instantaneous rate of change of  $W$  is equal to the average rate of change of  $W$  over the interval  $5 \leq v \leq 60$ .
- (c) Over the time interval  $0 \leq t \leq 4$  hours, the air temperature is a constant  $32^{\circ}\text{F}$ . At time  $t = 0$ , the wind velocity is  $v = 20$  mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at  $t = 3$  hours? Indicate units of measure.
- 

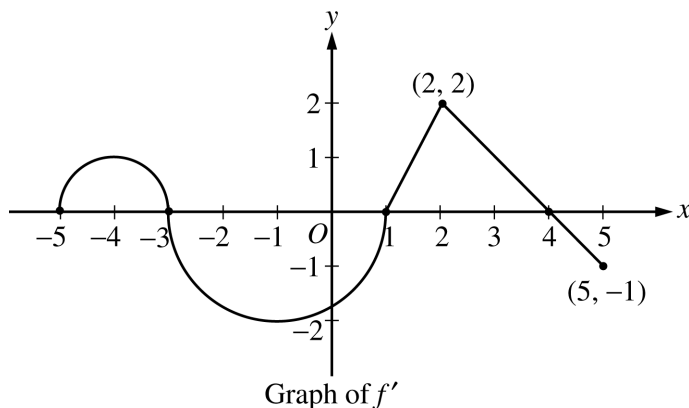
**WRITE ALL WORK IN THE EXAM BOOKLET.**

**END OF PART A OF SECTION II**

**2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

**CALCULUS AB  
SECTION II, Part B  
Time—45 minutes  
Number of problems—3**

No calculator is allowed for these problems.



4. Let  $f$  be a function defined on the closed interval  $-5 \leq x \leq 5$  with  $f(1) = 3$ . The graph of  $f'$ , the derivative of  $f$ , consists of two semicircles and two line segments, as shown above.
- (a) For  $-5 < x < 5$ , find all values  $x$  at which  $f$  has a relative maximum. Justify your answer.
  - (b) For  $-5 < x < 5$ , find all values  $x$  at which the graph of  $f$  has a point of inflection. Justify your answer.
  - (c) Find all intervals on which the graph of  $f$  is concave up and also has positive slope. Explain your reasoning.
  - (d) Find the absolute minimum value of  $f(x)$  over the closed interval  $-5 \leq x \leq 5$ . Explain your reasoning.

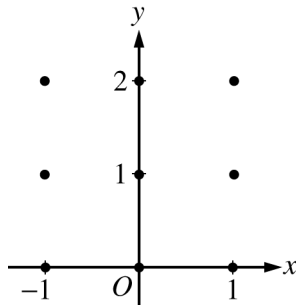
**WRITE ALL WORK IN THE EXAM BOOKLET.**

**2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

5. Consider the differential equation  $\frac{dy}{dx} = \frac{1}{2}x + y - 1$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ . Describe the region in the  $xy$ -plane in which all solution curves to the differential equation are concave up.

(c) Let  $y = f(x)$  be a particular solution to the differential equation with the initial condition  $f(0) = 1$ . Does  $f$  have a relative minimum, a relative maximum, or neither at  $x = 0$ ? Justify your answer.

(d) Find the values of the constants  $m$  and  $b$ , for which  $y = mx + b$  is a solution to the differential equation.

6. Let  $f$  be a twice-differentiable function such that  $f(2) = 5$  and  $f(5) = 2$ . Let  $g$  be the function given by  $g(x) = f(f(x))$ .

(a) Explain why there must be a value  $c$  for  $2 < c < 5$  such that  $f'(c) = -1$ .

(b) Show that  $g'(2) = g'(5)$ . Use this result to explain why there must be a value  $k$  for  $2 < k < 5$  such that  $g''(k) = 0$ .

(c) Show that if  $f''(x) = 0$  for all  $x$ , then the graph of  $g$  does not have a point of inflection.

(d) Let  $h(x) = f(x) - x$ . Explain why there must be a value  $r$  for  $2 < r < 5$  such that  $h(r) = 0$ .

**WRITE ALL WORK IN THE EXAM BOOKLET.**

**END OF EXAM**